8.1.1 EXERCISES

For a link to all of the additional resources available for this section, click OSttS Chapter 8 materials. For help with Exercises 1 - 26, click on one or more of the resources below:

- Solving a system of linear equations using substitution
- Solving a system of linear equations using elimination (consistent independent systems)
- Solving a system of linear equations using elimination (consistent dependent systems)

(Review Exercises) In Exercises 1 - 8, take a trip down memory lane and solve the given system using substitution and/or elimination. Classify each system as consistent independent, consistent dependent, or inconsistent. Check your answers both algebraically and graphically.

1. $\begin{cases} x + 2y = 5 \\ x = 6 \end{cases}$

2. $\begin{cases} 2y - 3x = 1 \\ y = -3 \end{cases}$

3. $\begin{cases} \frac{x+2y}{4} = -5 \\ \frac{3x-y}{2} = 1 \end{cases}$

4. $\begin{cases} \frac{2}{3}x - \frac{1}{5}y = 3 \\ \frac{1}{2}x + \frac{3}{4}y = 1 \end{cases}$

5. $\begin{cases} \frac{1}{2}x - \frac{1}{3}y = -1 \\ 2y - 3x = 6 \end{cases}$

6. $\begin{cases} x + 4y = 6 \\ \frac{1}{12}x + \frac{1}{3}y = \frac{1}{2} \end{cases}$

7. $\begin{cases} 3y - \frac{3}{2}x = -\frac{15}{2} \\ \frac{1}{2}x - y = \frac{3}{2} \end{cases}$

8. $\begin{cases} \frac{5}{6}x + \frac{5}{3}y = -\frac{7}{3} \\ -\frac{10}{3}x - \frac{20}{3}y = 10 \end{cases}$

In Exercises 9 - 26, put each system of linear equations into triangular form and solve the system if possible. Classify each system as consistent independent, consistent dependent, or inconsistent.

9. $\begin{cases} -5x + y = 17 \\ x + y = 5 \end{cases}$

10. $\begin{cases} x + y + z = 3 \\ 2x - y + z = 0 \\ -3x + 5y + 7z = 7 \end{cases}$

11. $\begin{cases} 4x - y + z = 5 \\ 2y + 6z = 30 \\ x + z = 5 \end{cases}$

12. $\begin{cases} 4x - y + z = 5 \\ 2y + 6z = 30 \\ x + z = 6 \end{cases}$

13. $\begin{cases} x + y + z = -17 \\ y - 3z = 0 \end{cases}$

14. $\begin{cases} x - 2y + 3z = 7 \\ -3x + y + 2z = -5 \\ 2x + 2y + z = 3 \end{cases}$

15. $\begin{cases} 3x - 2y + z = -5 \\ x + 3y - z = 12 \\ x + y + 2z = 0 \end{cases}$

16. $\begin{cases} 2x - y + z = -1 \\ 4x + 3y + 5z = 1 \\ 5y + 3z = 4 \end{cases}$

17. $\begin{cases} x - y + z = -4 \\ -3x + 2y + 4z = -5 \\ x - 5y + 2z = -18 \end{cases}$

18. $\begin{cases} 2x - 4y + z = -7 \\ x - 2y + 2z = -2 \\ -x + 4y - 2z = 3 \end{cases}$

19. $\begin{cases} 2x - y + z = 1 \\ 2x + 2y - z = 1 \\ 3x + 6y + 4z = 9 \end{cases}$

20. $\begin{cases} x - 3y - 4z = 3 \\ 3x + 4y - z = 13 \\ 2x - 19y - 19z = 2 \end{cases}$

21. $\begin{cases} x + y + z = 4 \\ 2x - 4y - z = -1 \\ x - y = 2 \end{cases}$

22. $\begin{cases} x - y + z = 8 \\ 3x + 3y - 9z = -6 \\ 7x - 2y + 5z = 39 \end{cases}$

23. $\begin{cases} 2x - 3y + z = -1 \\ 4x - 4y + 4z = -13 \\ 6x - 5y + 7z = -25 \end{cases}$

24. $\begin{cases} 2x_1 + x_2 - 12x_3 - x_4 = 16 \\ -x_1 + x_2 + 12x_3 - 4x_4 = -5 \\ 3x_1 + 2x_2 - 16x_3 - 3x_4 = 25 \\ x_1 + 2x_2 - 5x_4 = 11 \end{cases}$

25. $\begin{cases} x_1 - x_3 = -2 \\ 2x_2 - x_4 = 0 \\ x_1 - 2x_2 + x_3 = 0 \\ -x_3 + x_4 = 1 \end{cases}$

26. $\begin{cases} x_1 - x_2 - 5x_3 + 3x_4 = -1 \\ x_1 + x_2 + 5x_3 - 3x_4 = 0 \\ x_2 + 5x_3 - 3x_4 = 1 \\ x_1 - 2x_2 - 10x_3 + 6x_4 = -1 \end{cases}$

27. Find two other forms of the parametric solution to Exercise 11 above by reorganizing the equations so that x or y can be the free variable.

For help with Exercises 28 - 34, click on one or more of the resources below:

- Solving 'mixture' problems
- Solving a system of linear equations by elimination and an application

28. A local buffet charges \$7.50 per person for the basic buffet and \$9.25 for the deluxe buffet (which includes crab legs.) If 27 diners went out to eat and the total bill was \$227.00 before taxes, how many chose the basic buffet and how many chose the deluxe buffet?

29. At The Old Home Fill'er Up and Keep on a-Truckin' Cafe, Mavis mixes two different types of coffee beans to produce a house blend. The first type costs \$3 per pound and the second costs \$8 per pound. How much of each type does Mavis use to make 50 pounds of a blend which costs \$6 per pound?

30. Skippy has a total of \$10,000 to split between two investments. One account offers 3% simple interest, and the other account offers 8% simple interest. For tax reasons, he can only earn \$500 in interest the entire year. How much money should Skippy invest in each account to earn \$500 in interest for the year?

31. A 10% salt solution is to be mixed with pure water to produce 75 gallons of a 3% salt solution. How much of each are needed?

32. At The Crispy Critter's Head Shop and Patchouli Emporium along with their dried up weeds, sunflower seeds and astrological postcards they sell an herbal tea blend. By weight, Type I herbal tea is 30% peppermint, 40% rose hips and 30% chamomile, Type II has percents 40%, 20% and 40%, respectively, and Type III has percents 35%, 30% and 35%, respectively. How much of each Type of tea is needed to make 2 pounds of a new blend of tea that is equal parts peppermint, rose hips and chamomile?

33. Discuss with your classmates how you would approach Exercise 32 above if they needed to use up a pound of Type I tea to make room on the shelf for a new canister.

34. If you were to try to make 100 mL of a 60% acid solution using stock solutions at 20% and 40%, respectively, what would the triangular form of the resulting system look like? Explain.

Checkpoint Quiz 8.1

1. Consider the system: $\begin{cases} 2x - 3y + z &= 2 \\ 3x - y + 2z &= -1 \\ 7x + 5z &= -5 \end{cases}$

   (a) Put the following system in triangular form and solve.

   (b) Is this system consistent or inconsistent? If consistent, is it dependent or independent?

   (c) Check your answer algebraically.

For worked out solutions to this quiz, click the links below:

- Quiz Solution Part 1
- Quiz Solution Part 2

8.1.2 Answers

1. Consistent independent
   Solution $\left(6, -\frac{1}{2}\right)$

2. Consistent independent
   Solution $\left(-\frac{7}{3}, -3\right)$

3. Consistent independent
   Solution $\left(-\frac{16}{7}, -\frac{62}{7}\right)$

4. Consistent independent
   Solution $\left(\frac{49}{12}, -\frac{25}{18}\right)$

5. Consistent dependent
   Solution $\left(t, \frac{3}{2}t + 3\right)$
   for all real numbers t

6. Consistent dependent
   Solution $(6 - 4t, t)$
   for all real numbers t

7. Inconsistent
   No solution

8. Inconsistent
   No solution

Because triangular form is not unique, we give only one possible answer to that part of the question. Yours may be different and still be correct.

9. $\begin{cases} x + y = 5 \\ y = 7 \end{cases}$
   Consistent independent
   Solution $(-2, 7)$

10. $\begin{cases} x - \frac{5}{3}y - \frac{7}{3}z = -\frac{7}{3} \\ y + \frac{5}{4}z = 2 \\ z = 0 \end{cases}$
    Consistent independent
    Solution $(1, 2, 0)$

11. $\begin{cases} x - \frac{1}{4}y + \frac{1}{4}z = \frac{5}{4} \\ y + 3z = 15 \\ 0 = 0 \end{cases}$
    Consistent dependent
    Solution $(-t + 5, -3t + 15, t)$
    for all real numbers t

12. $\begin{cases} x - \frac{1}{4}y + \frac{1}{4}z = \frac{5}{4} \\ y + 3z = 15 \\ 0 = 1 \end{cases}$
    Inconsistent
    No solution

13. $\begin{cases} x + y + z = -17 \\ y - 3z = 0 \end{cases}$
    Consistent dependent
    Solution $(-4t - 17, 3t, t)$
    for all real numbers t

14. $\begin{cases} x - 2y + 3z = 7 \\ y - \frac{11}{5}z = -\frac{16}{5} \\ z = 1 \end{cases}$
    Consistent independent
    Solution $(2, -1, 1)$

15. $\begin{cases} x+y+2z = 0 \\ y-\frac{3}{2}z = 6 \\ z = -2 \end{cases}$ Consistent independent
Solution $(1,3,-2)$

16. $\begin{cases} x-\frac{1}{2}y+\frac{1}{2}z = -\frac{1}{2} \\ y+\frac{3}{5}z = \frac{3}{5} \\ 0 = 1 \end{cases}$ Inconsistent
no solution

17. $\begin{cases} x-y+z = -4 \\ y-7z = 17 \\ z = -2 \end{cases}$ Consistent independent
Solution $(1,3,-2)$

18. $\begin{cases} x-2y+2z = -2 \\ y = \frac{1}{2} \\ z = 1 \end{cases}$ Consistent independent
Solution $\left(-3,\frac{1}{2},1\right)$

19. $\begin{cases} x-\frac{1}{2}y+\frac{1}{2}z = \frac{1}{2} \\ y-\frac{2}{3}z = 0 \\ z = 1 \end{cases}$ Consistent independent
Solution $\left(\frac{1}{3},\frac{2}{3},1\right)$

20. $\begin{cases} x-3y-4z = 3 \\ y+\frac{11}{13}z = \frac{4}{13} \\ 0 = 0 \end{cases}$ Consistent dependent
Solution $\left(\frac{19}{13}t+\frac{51}{13},-\frac{11}{13}t+\frac{4}{13},t\right)$
for all real numbers t

21. $\begin{cases} x+y+z = 4 \\ y+\frac{1}{2}z = \frac{3}{2} \\ 0 = 1 \end{cases}$ Inconsistent
no solution

22. $\begin{cases} x-y+z = 8 \\ y-2z = -5 \\ z = 1 \end{cases}$ Consistent independent
Solution $(4,-3,1)$

23. $\begin{cases} x-\frac{3}{2}y+\frac{1}{2}z = -\frac{1}{2} \\ y+z = -\frac{11}{2} \\ 0 = 0 \end{cases}$ Consistent dependent
Solution $\left(-2t-\frac{35}{4},-t-\frac{11}{2},t\right)$
for all real numbers t

24. $\begin{cases} x_1+\frac{2}{3}x_2-\frac{16}{3}x_3-x_4 = \frac{25}{3} \\ x_2+4x_3-3x_4 = 2 \\ 0 = 0 \\ 0 = 0 \end{cases}$ Consistent dependent
Solution $(8s-t+7,-4s+3t+2,s,t)$
for all real numbers s and t

25. $\begin{cases} x_1-x_3 = -2 \\ x_2-\frac{1}{2}x_4 = 0 \\ x_3-\frac{1}{2}x_4 = 1 \\ x_4 = 4 \end{cases}$ Consistent independent
Solution $(1,2,3,4)$

26. $\begin{cases} x_1 - x_2 - 5x_3 + 3x_4 = -1 \\ x_2 + 5x_3 - 3x_4 = \frac{1}{2} \\ 0 = 1 \\ 0 = 0 \end{cases}$ Inconsistent
No solution

27. If x is the free variable then the solution is $(t, 3t, -t+5)$ and if y is the free variable then the solution is $\left(\frac{1}{3}t, t, -\frac{1}{3}t+5\right)$.

28. 13 chose the basic buffet and 14 chose the deluxe buffet.

29. Mavis needs 20 pounds of \$3 per pound coffee and 30 pounds of \$8 per pound coffee.

30. Skippy needs to invest \$6000 in the 3% account and \$4000 in the 8% account.

31. 22.5 gallons of the 10% solution and 52.5 gallons of pure water.

32. $\frac{4}{3} - \frac{1}{2}t$ pounds of Type I, $\frac{2}{3} - \frac{1}{2}t$ pounds of Type II and t pounds of Type III where $0 \leq t \leq \frac{4}{3}$.